

Gauge field production in SUGRA inflation: local non-Gaussianity and primordial black holes

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When inflation is driven by a pseudo-scalar field χ coupled to vectors as $\frac{\alpha}{4} \chi F \tilde{F}$, this coupling may lead to a copious production of gauge quanta, which in turns induces non-Gaussian and non-scale invariant corrections to curvature perturbations. We point out that this mechanism is generically at work in a broad class of inflationary models in supergravity hence providing them with a rich set of observational predictions. When the gauge fields are massless, significant effects on CMB scales emerge only for relatively large α . We show that in this regime, the curvature perturbations produced at the last stages of inflation have a relatively large amplitude that is of the order of the upper bound set by the possible production of primordial black holes by non-Gaussian perturbations. On the other hand, within the supergravity framework described in our paper, the gauge fields can often acquire a mass through a coupling to additional light scalar fields. Perturbations of these fields modulate the duration of inflation, which serves as a source for non-Gaussian perturbations of the metric. In this regime, the bounds from primordial black holes are parametrically satisfied and non-Gaussianity of the local type can be generated at the observationally interesting level $f_{\text{NL}} \sim \mathcal{O}(10 - 100)$.

I. INTRODUCTION

In a recent series of papers [1–3] a new broad class of models of chaotic inflation in supergravity has been developed. These models generalize the simplest model of this type proposed long ago in [4]; see also [5–20] for a partial list of other closely related publications.

The new class of models [1–3] describes two scalar fields, S and Φ , with the superpotential

$$W = S f(\Phi) , \quad (1)$$

where $f(\Phi)$ is a real holomorphic function such that $\bar{f}(\bar{\Phi}) = f(\Phi)$. Any function which can be represented by Taylor series with real coefficients has this property. The Kähler potential can be chosen to have functional form

$$K = K((\Phi - \bar{\Phi})^2, S\bar{S}). \quad (2)$$

In this case, the Kähler potential does not depend on $\phi = \sqrt{2} \text{Re } \Phi$. Under certain conditions on the Kähler potential, inflation occurs along the direction $S = \text{Im } \Phi = 0$, and the field ϕ plays the role of the inflaton field with the potential

$$V(\phi) = |f(\phi/\sqrt{2})|^2. \quad (3)$$

All scalar fields have canonical kinetic terms along the inflationary trajectory $S = \text{Im } \Phi = 0$.

This class of models can be further extended [3, 11] to incorporate a KKLT-type construction with strong moduli stabilization [21], which may have interesting phenomenological consequences and may provide a simple solution of the cosmological moduli and gravitino problems [22, 23].

The generality of the functional form of the inflationary potential $V(\phi)$ allows one to describe *any* combination of the parameters n_s and r . Thus, this rather simple class of models may describe *any* set of observational data which can be expressed in terms of these two parameters by an appropriate choice of the function $f(\Phi)$ in the superpotential. Meanwhile the choice of the Kähler potential controls masses of the fields orthogonal to the inflationary trajectory [1–3]. Reheating in this scenario requires considering the scalar-vector coupling $\sim \phi F_{\mu\nu} F^{\mu\nu}$ [3, 24]. If not only the inflaton but some other scalar field has a mass much smaller than H during inflation, one may use it as a curvaton field [25] for generation of non-Gaussian perturbations in this class of models [26].

In this paper, we will study an alternative mechanism of production of non-Gaussian perturbations, using another formulation of this class of models, with

$$K = K((\Phi + \bar{\Phi})^2, S\bar{S}). \quad (4)$$

In this case, the Kähler potential does not depend on $\chi = \sqrt{2} \text{Im } \Phi$, which plays the role of the inflaton field with the potential

$$V(\chi) = |f(\chi/\sqrt{2})|^2. \quad (5)$$

The difference is that now the inflaton field is a pseudo scalar, which can have a coupling to vector fields

$$\frac{\alpha}{4} \chi F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (6)$$

where $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ and α is a dimensionful constant. This coupling is expected to be present since it is compatible with all the symmetries, including a shift-symmetry in χ .

The study of the phenomenological effects of such a coupling during inflation has received a lot of attention lately [27–33]. In particular, it has been shown in [30, 31] that, if the constant α is large enough, such a coupling can lead to a copious production of gauge fields due to the time dependence of χ . Through their *inverse decay* into inflaton perturbations, these gauge fields yield an additional contribution to the scalar power spectrum which is both non-Gaussian and violates scale invariance. In this way it is possible to obtain non-Gaussian and non-scale invariant effects that can be observed by the Planck satellite and has not yet been ruled out yet by WMAP, although the parameter space corresponding to such a signal is relatively small [33]. In addition, gauge fields source tensor modes and lead to a stochastic gravity wave signal that could be detected at interferometers such as Advanced LIGO or Virgo [32, 34] (see also [35]).

Since the new class of inflationary models in supergravity needs a coupling between the inflaton and gauge fields to have successful reheating, we have to consistently take into account the violations of Gaussianity and scale invariance induced by the inverse decay mechanism. This is the topic of section II.

A potential threat in this model is the overproduction of primordial black holes. As we will see in section III, at very small scales, far beyond what is observable by the CMB, the produced gauge quanta largely increase the curvature power spectrum. At some point, various forms of backreaction stops this growth, but by then the power spectrum has reached $\Delta_\zeta^2 \sim \mathcal{O}(10^{-3})$. At such high values, a statistical fluctuation might locally increase the density so that primordial black holes are formed. In this way the non-detection of primordial black holes puts an observational upper bound on the power spectrum [36–41], which we discuss in section IV. Our estimates for the late-time power spectrum land a factor of six above this bound (compare e.g. (33) with (39)). Since we expect our estimate to be reliable up to factors of order one, we cannot definitively claim that the inverse decay mechanism and its interesting phenomenology is incompatible with current data, but our result on production of primordial black holes highlights a clear tension.

In section V we argue that in supergravity inflationary models the gauge field can naturally acquire a mass through the Higgs mechanism and as consequence of this

local non-Gaussianity in the observationally interesting region $f_{NL}^{loc} \sim 10 - 100$ can be produced via the mechanism pointed out in [33]. Consider the additional scalar field σ with a small mass $m_\sigma \ll H$ during inflation that gives mass to the gauge fields via spontaneous symmetry breaking. The light field σ and, correspondingly, the vector field mass, experiences inflationary fluctuations. In the parts of the universe where the value of the vector field mass is small, the vector field fluctuations are easily produced since the gauge mass quenches the tachyonic instability. This in turns leads to a longer stage of inflation because of the additional friction generated by the gauge fields. Meanwhile in the parts of the universe where the fluctuations of the light scalar field σ make this field large, the vector field mass becomes larger and inflation is shorter due to the lack of backreaction. As a result, fluctuations of the light scalar field σ lead to fluctuations of the total number of e-foldings δN , i.e. to adiabatic perturbations of metric. This effect may generate significant primordial local non-Gaussianity without the need to require a complicated regime of reheating which is necessary in the conventional curvaton scenario. Also, in the regime of parameters relevant for this scenario the primordial black hole bounds are satisfied parametrically.

One might suspect that this regime is not very natural, as it requires additional fine-tuning of the theory in order to introduce the mass with $m_\sigma \ll H$. However, as we are going to show, in the context of the supergravity models [1–4] this regime emerges in a very natural way. One may consider, for example, the supercurvaton model of Ref. [26], where the role of the curvaton field σ is played by the radial component of the light complex field S , and generalize this model by coupling the field S to the vector field. In that case, the vector field acquires the mass proportional to σ , which experiences inflationary fluctuations.

In section VII we study the evolution of the light field σ during inflation in our scenario, which is similar to the evolution of the curvaton field σ in [26], so we will continue calling this field the curvaton, and use the results of [26] for the description of its evolution. In the original model of [26], just as in any other curvaton model [25], adiabatic perturbations of metric are generated by perturbations of the field σ after a complicated sequence of reheating, expansion of the universe, and the subsequent decay of the curvaton field. In our scenario, adiabatic perturbations are produced due to the modulation of the duration of inflation by the perturbations of the field σ . As we will demonstrate, this mechanism can easily produce local non-Gaussianity in the potentially interesting range f_{NL} from $\mathcal{O}(10)$ to $\mathcal{O}(100)$, even if the coupling constant α is not very large.

Finally, in section VIII, we find that typical values of the coupling constant α considered in this work lead to a relatively high perturbative reheating temperature $T \sim 10^{10} \text{ GeV}$. This should be read as a lower limit, since

the copious non-perturbative production of gauge fields already during inflation could lead to and even higher reheating temperature. This could lead to the cosmological gravitino problem [42], but in the class of models with strong moduli stabilization and gravitino mass $\mathcal{O}(100)$ TeV this problem does not appear [23].

II. CMB SCALES: VIOLATIONS OF GAUSSIANITY AND SCALE INVARIANCE

Recently there has been a lot of interest in the effect of gauge field production in axion inflation [27–33]. In this section we summarize the main points.

Consider a pseudo-scalar inflaton with a potential suitable for inflation. The symmetries of the theory allow for a coupling $\chi F_{\mu\nu} \tilde{F}^{\mu\nu}$ to some $U(1)$ gauge sector. This coupling is essential for reheating in the supergravity models we discussed in section I. We will therefore consider the following bosonic part of the action¹

$$S = - \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial\chi)^2 + \frac{1}{4} F^2 + \frac{\alpha}{4} \chi F \tilde{F} + V(\chi) \right].$$

Since all relevant effects arise from the couplings above we can safely neglect the gravitational interaction between perturbations and work with an unperturbed FLRW metric². We organize the perturbation theory based on the equations that we are able to solve. Consider two classical³ fields $\vec{A}(x, t)$ and $\chi(t)$ that solve these two coupled differential equations

$$\ddot{\chi} + 3H\dot{\chi} + V'(\chi) = \alpha \langle \vec{E} \cdot \vec{B} \rangle, \quad (7)$$

$$\vec{A}'' - \nabla^2 \vec{A} - \alpha \chi' \nabla \times \vec{A} = 0, \quad (8)$$

where $\vec{E} \equiv -\dot{\vec{A}}/a$, $\vec{B} \equiv a^{-2} \nabla \times \vec{A}$ and $\vec{E} \cdot \vec{B} = -F\tilde{F}/4$ are computed from \vec{A} .

Now let us look at the action expanded around χ and \vec{A} , i.e. $S[\chi + \delta\chi, \vec{A} + \delta\vec{A}]$. Organizing the result at various

orders in $\delta\chi$ and $\delta\vec{A}$ one finds

$$\begin{aligned} S = \text{const} - \int d^4x \sqrt{-g} (\delta\chi) \alpha \left[\langle \vec{E} \cdot \vec{B} \rangle - \vec{E} \cdot \vec{B} \right] \quad (9) \\ - \int d^4x \sqrt{-g} \left[\frac{1}{2} (\partial\delta\chi)^2 + \frac{1}{2} V''(\chi) (\delta\chi)^2 + \frac{1}{4} (\delta F)^2 \right. \\ \left. + \frac{\alpha}{4} \chi \delta F \delta \tilde{F} + \frac{\alpha}{2} \delta\chi \delta F \tilde{F} \right] \\ - \int d^4x \sqrt{-g} \left[\frac{\alpha}{4} \delta\chi \delta F \delta \tilde{F} + \frac{1}{6} (\delta\chi)^3 V'''(\chi) \right], \end{aligned}$$

where again the classical background fields χ and \vec{A} solve (7) and (8). Notice that there is a “tadpole” for $\delta\chi$ due to the fact that at the background level we solved an inhomogeneous equation for \vec{A} but just a homogeneous one for χ . From this term one also sees that $\delta\chi$ will source δA^0 , hence it will modify the constraint. The equations of motion in Coulomb gauge $\partial_i A^i = 0$ are

$$a\delta\ddot{A}_i - \frac{\partial_k^2(\delta A_i)}{a} + aH\dot{\delta A}_i - \alpha\dot{\chi}\nabla \times (\delta\vec{A}) = \quad (10)$$

$$\alpha\dot{\chi}\nabla \times \vec{A} - \alpha(\nabla\delta\chi) \times \vec{A} - \partial_t(a\partial_i(\delta A^0)), \quad (11)$$

$$\begin{aligned} (\delta\ddot{\chi}) + 3H\dot{\delta\chi} - \nabla^2\delta\chi + V''\delta\chi = \\ \frac{\alpha}{4} \left(\langle F\tilde{F} \rangle - F\tilde{F} - 2F\tilde{F} \right), \quad (12) \\ a\partial_i\partial_i(\delta A^0) = -\alpha\nabla(\delta\chi) \cdot \nabla \times \vec{A}. \end{aligned}$$

The solution for the constraint equation for δA^0 is

$$\delta A^0(x, t) = a^{-1} \int d^3y \frac{\alpha\nabla(\delta\chi) \cdot \nabla \times \vec{A}}{4\pi|x-y|}. \quad (13)$$

Unfortunately this coupled system of equations is hard to solve. Hence [30, 31] made the approximation of neglecting all terms quadratic or higher in $\delta\chi$, δA and A . This is a good approximation as long as $F\tilde{F}$ (or equivalently $\langle \vec{E} \cdot \vec{B} \rangle$) is not too large (a more quantitative condition is given in (29)), which is the regime we will discuss in this section. In the next section we will see that, since $\langle \vec{E} \cdot \vec{B} \rangle$ grows with time, towards the end of inflation this description is not valid anymore, and one has to take backreaction into account.

Solving the approximated equations of motion

$$a\ddot{A}_i - \frac{\partial_k^2 A_i}{a} + aH\dot{A}_i - \alpha\dot{\chi}\nabla \times \vec{A} = 0 \quad (14)$$

$$\delta\ddot{\chi} + 3H\dot{\delta\chi} - \nabla^2\delta\chi + V''\delta\chi = \alpha \left(\langle \vec{E} \cdot \vec{B} \rangle - \vec{E} \cdot \vec{B} \right) \quad (15)$$

one finds a tachyonic enhancement of the gauge fields. For the growing mode of one of the two polarizations of the gauge field we get

$$A = \frac{1}{\sqrt{2k}} e^{\pi\xi/2} W_{-i\xi, 1/2}(2ik\tau). \quad (16)$$

¹ Notice that in the existing literature, such a coupling is usually associated with interaction of the axion field with vector fields, with a coupling $-\frac{\alpha}{4f}$. In our approach it is not necessary to associate the pseudo scalar field with the axion field with the radius of the potential $\sim f$, so we normalize the coupling in terms of the reduced Planck mass M_p , which we then set to one, and consider the following interaction term $-\frac{\alpha}{4}\chi F_{\mu\nu} \tilde{F}^{\mu\nu}$.

² We are neglecting vector and tensor modes and the slow-roll suppressed interactions coming from the solution of the constraints on the lapse and the shift.

³ Here we are assuming that the occupation number of the relevant gauge modes is large enough that one can approximate the resulting electromagnetic field with a classical one. This assumption is implicit in all other approaches so far.

Here $W_{\lambda,\mu}(x)$ denotes the Whittaker function, and ξ is defined as⁴

$$\xi \equiv -\frac{\dot{\chi}\alpha}{2H}. \quad (17)$$

As we see, the relation between the coupling constant α and the value of ξ 60 e-foldings before the end of inflation is model dependent, but for our model there is an approximate relation which is valid for the parameters that we are going to explore:

$$\alpha \sim 15\xi. \quad (18)$$

For $\xi > 1$ the new coupling therefore leads to generation of perturbations of the vector fields around horizon scales. The produced gauge fields then change the dynamics of χ and H . The cosmological homogeneous Klein-Gordon equation and the Friedmann equation get extra contributions from the gauge fields and can now be written as

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial \chi} = \alpha \langle \vec{E} \cdot \vec{B} \rangle \quad (19)$$

$$3H^2 = \frac{1}{2}\dot{\chi}^2 + V + \frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle. \quad (20)$$

They are computed as

$$\langle \vec{E} \cdot \vec{B} \rangle = \frac{1}{4\pi^2 a^4} \int_0^\infty dk k^3 \frac{\partial}{\partial \tau} |A|^2, \quad (21)$$

$$\langle \frac{\vec{E}^2 + \vec{B}^2}{2} \rangle = \frac{1}{4\pi^2 a^4} \int_0^\infty dk k^2 \left[|A'|^2 + k^2 |A|^2 \right]. \quad (22)$$

After renormalization, one can reduce the integration interval to the region $\frac{1}{8\xi} < \frac{k}{aH} < 2\xi$, which is where the enhancement in the (derivative of the) gauge field takes place.

From the homogeneous Klein-Gordon equation (15) one reads off that the influence of the produced gauge fields on the homogeneous dynamics of χ and H can be safely neglected as long as

$$\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}} \ll 1, \quad \frac{\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle}{3H^2} \ll 1. \quad (23)$$

⁴ Note that we have some minus signs different from [30], but this is a matter of conventions. We will work with a model that has $\dot{\chi} < 0$ during inflation and define ξ to be positive. The sign of $\langle \vec{E} \cdot \vec{B} \rangle$ is always opposite to the sign of $\dot{\chi}$. Therefore the physical effect of the tachyonic enhancement is always that inflation is prolonged. To be precise: when $\dot{\chi}$ is negative, the growing field is actually the opposite polarization, i.e. A_- , which makes that $\langle \vec{E} \cdot \vec{B} \rangle > 0$ (see, for example, equation (8) in [28]).

Of these two conditions the first one is always the most stringent. When it stops to hold, backreaction on the homogeneous evolution becomes important and the evolution of χ and H will be slowed down, which makes inflation lasts longer. We will see in the next section that backreaction on the inhomogeneous equation for $\delta\chi$ happens even earlier. In this section we focus on the regime in which all of these effects are negligible, which e.g. for a quadratic potential corresponds roughly to $\xi \lesssim 4$. This is appropriate for the description of CMB scales.

Now we move to the power spectrum. The copiously generated gauge fields may, by inverse decay, produce additional perturbations of the inflaton field $\delta\chi$, proportional to the square of the vector field perturbations. As was shown in [30, 31], this can be described (up to backreaction effects to be described in the next section) by using (15). The inclusion of the source term leads to an extra contribution to the power spectrum of the curvature perturbation on uniform density hypersurfaces $\zeta = -\frac{H}{\dot{\chi}}\delta\chi$, which has been computed in [30, 31] (we present a quick estimate in Appendix (B))

$$\Delta_\zeta^2(k) = \Delta_{\zeta,\text{sr}}^2(k) (1 + \Delta_{\zeta,\text{sr}}^2(k) f_2(\xi) e^{4\pi\xi}), \quad (24)$$

where $f_2(\xi)$ was defined in [30, 31] and can be computed numerically (a useful large ξ approximation is given in (B14)) and

$$\Delta_{\zeta,\text{sr}}^2(k) = \left(\frac{H^2}{2\pi|\dot{\chi}|} \right)^2 \quad (25)$$

is the amplitude of the vacuum inflationary perturbations as in standard slow-roll inflation. WMAP [43] has measured $\Delta_{\zeta,\text{sr}}^2(k_\star) = 2.43 \cdot 10^{-9}$, where $k_\star = 0.002\text{Mpc}^{-1}$ is the pivot scale that we will take to correspond with $N = 60$ e-foldings before the end of inflation. The second term in (24) violates both scale invariance (and Gaussianity as we will see below), since it comes schematically from $(\delta A)^2$, i.e. the square of a Gaussian which grows with time as in (16).

We move to the bispectrum. The produced gauge fields lead to equilateral non-Gaussianity in the CMB [30, 31]

$$f_{NL} = \frac{\Delta_\zeta^6(k)}{\Delta_{\zeta,\text{sr}}^4(k)} e^{6\pi\xi} f_3(\xi), \quad (26)$$

where $f_3(\xi)$ another function defined in [30, 31], which can be computed numerically (see (D7) for a useful approximation). The amount of non-Gaussianity, therefore, depends exponentially on ξ . Between $\xi = 0$ and $\xi = 3$ it grows from $\mathcal{O}(1)$ to $\mathcal{O}(10^4)$ and most of the growth takes place in a small interval around $\xi \simeq 2.5$.

The analysis of [33] showed that the bounds coming from the power spectrum (especially from WMAP plus

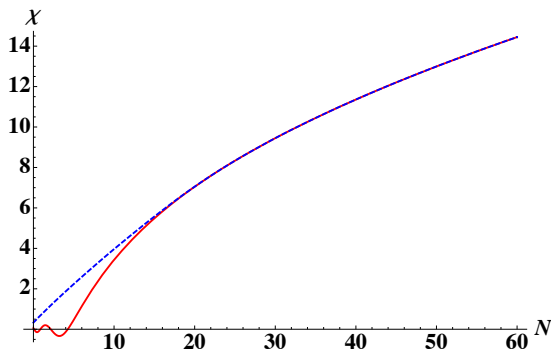


FIG. 1. The evolution of the inflaton field χ , as a function of the number of e-folds N left to the end of inflation (time is moving to the left) for $\xi[N = 60] = 2.2$. The result in dashed blue does take backreaction from the sources in equations (19) and (20) into account, the result in red does not. It is clear that backreaction prolongs inflation.

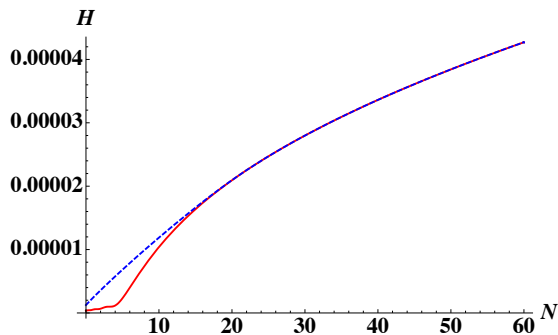


FIG. 2. The evolution of the Hubble scale H as a function of N for $\xi[N = 60] = 2.2$. Again the dashed blue line is the result corrected for backreaction from the sources in equations (19) and (20).

ACT, because of the violation of scale invariance) and from the bispectrum (from WMAP) are compatible, with the former being typically slightly more stringent. Specifying a confidence region in ξ requires assuming some prior for this parameter. The physically best motivated prior is log-flat in ξ reflecting the fact the scale of the dimension five coupling $\chi F\tilde{F}$ could be anywhere (with strong indications that it should be below the Planck scale [44]). In this case at 95% CL one finds $\xi \lesssim 2.2$. A flat prior on ξ leads to $\xi \lesssim 2.4$.

III. VERY SMALL SCALES: STRONG BACKREACTION

In this section we want to estimate the power spectrum and bispectrum towards the end of inflation, i.e. on scales that are too small to be observed in the CMB. The only observational handle available in this regime is the non-detection of primordial black holes, which puts an upper

bound on the power spectrum [36–41].

To make these estimates it is essential to recognize that many of the formulae described in the previous section and given in the literature about inverse decay are valid only in the regime in which backreaction on the inhomogeneous equation for $\delta\chi$ is small (see (29)). As we show in the following, the scales relevant for the production of primordial black holes leave the horizon when backreaction is large. The authors of [45] did not account for backreaction and therefore their conclusion that gauge field production during inflation leads to black hole production might be premature.

For concreteness, we will consider a quadratic potential $V(\chi) = \frac{1}{2}m^2\chi^2$, with the mass chosen such that at the pivot scale k_* (that we take to correspond with $N = 60$) we get $\Delta_\zeta^2(k_*) = 2.43 \cdot 10^{-9}$.

Let us first look at the dynamics of χ and H . As we already discussed, when enough gauge field quanta have been produced, the conditions in (23) stop to hold (the inequality for $\langle \vec{E} \cdot \vec{B} \rangle$ is violated first) and χ and H are slowed down. As a result, inflation lasts longer. Let us check this. The behavior of χ , H and ξ as functions of N (the number of e-folds left to the end of inflation) follows from simultaneously solving (17), (19) and (20). In Figures (1) and (2) we have plotted the solutions for $\chi(N)$ and $H(N)$, with and without backreaction taken into account. For $\xi(N = 60) = 2.2$, the effect of backreaction becomes 10% around $N = 11$.

Now let us consider perturbations. Of course they will be affected by the backreaction on the homogeneous dynamics χ and H that we described above, but there is more. Let us consider (10)–(12). In the last section we solved for A in a *homogeneous* background and used that result (16) to compute the source term in the equation for χ perturbations. But as δA and $\delta\chi$ grow larger toward the end of inflation (both of them grow as $e^{2\pi\xi}$) the source in the right-hand side of (10) can not be neglected anymore. If we were able to solve this equation, we would find that $\vec{E} \cdot \vec{B}$ now depends on the perturbation $\delta\chi$. Expanding $\vec{E} \cdot \vec{B}$, which is the source term in (11), in powers of $\delta\chi$ we would find several new terms including additional friction and a modified speed of sound. In [28, 32] it was proposed how to estimate these effects in the regime of strong backreaction by just considering the additional friction term $\delta\chi$. The equation of motion for the perturbation $\delta\chi$ becomes

$$\delta\ddot{\chi} + 3\beta H \delta\dot{\chi} - \frac{\nabla^2}{a^2} \delta\chi + \frac{\partial^2 V}{\partial \chi^2} \delta\chi = \alpha [\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle], \quad (27)$$

with the additional friction term

$$\beta \equiv 1 - 2\pi\xi\alpha \frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}}. \quad (28)$$

Here the new term in β is caused by the dependence of $\langle \vec{E} \cdot \vec{B} \rangle$ on $\dot{\chi}$ (via its dependence on ξ). The behavior of β has been plotted in Figure (3). It is always positive⁵.

The new source of backreaction can be neglected as long as

$$2\pi\xi\alpha\frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}} \ll 1. \quad (29)$$

Note (from comparison with (23)) that the factor of $2\pi\xi$ makes that backreaction on the power spectrum will become significant before backreaction on H and χ does. For $\xi(N=60)=2.2$ we find that backreaction becomes of order 10% ($\beta=1.1$) at $N=22$.

The modified equation of motion (27) suggests that (as was already noted in [32], see also appendix (B)) we can estimate

$$\delta\chi \approx \frac{\alpha(\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle)}{3\beta H^2} \quad (30)$$

which leads to the power spectrum

$$\Delta_\zeta^2(k) \simeq \langle \zeta(x)^2 \rangle \simeq \left(\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3\beta H \dot{\chi}} \right)^2. \quad (31)$$

This estimate turns out to be particularly good in the regime in which we can check it, i.e. when $\xi \lesssim 4$ when the backreaction is negligible and we can compare with (24) ((see appendix (B))). This gives us confidence to use it also in the strong backreaction regime. It is easy to see that when backreaction becomes large, the second term in (28) dominates, and we end up with

$$\Delta_\zeta^2(k) \simeq \left(\frac{1}{2\pi\xi} \right)^2. \quad (32)$$

The estimate (31) for the power spectrum has been plotted in Figure (4) together with the formula (24), valid only when backreaction is negligible. Indeed, in the regime of strong backreaction the power spectrum asymptotes the estimate in (32). At the end of inflation we have $\xi \simeq 6.7$ (for $\xi(N=60)=2.2$), which gives

$$\Delta_\zeta^2(k) \simeq 7.5 \cdot 10^{-4}. \quad (33)$$

⁵ We work with negative $\dot{\chi}$ which yields positive $\langle \vec{E} \cdot \vec{B} \rangle$, while working with $\dot{\chi} > 0$ gives $\langle \vec{E} \cdot \vec{B} \rangle < 0$.

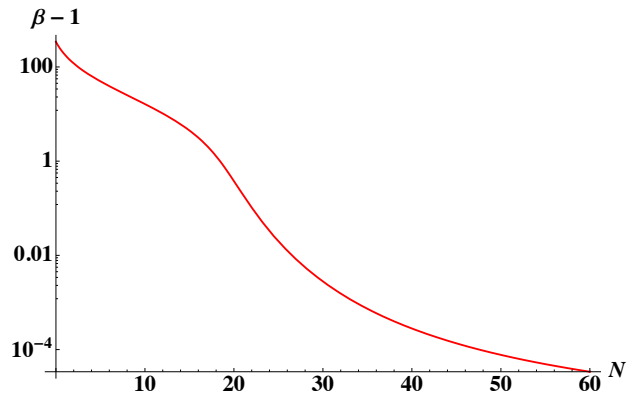


FIG. 3. Evolution of $(\beta-1)$ as function of N , for $\xi(N=60)=2.2$.

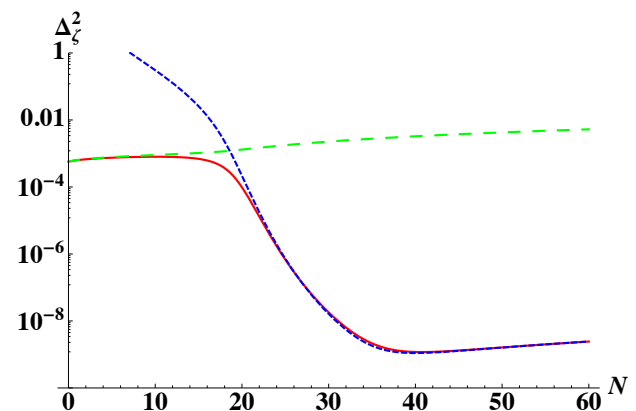


FIG. 4. Evolution of the power spectrum as function of N , for $\xi(N=60)=2.2$. The expression (24) that does not take backreaction into account is in finely dashed blue. In solid red is the estimate (31). When backreaction becomes significant this estimate coincides with the late-time estimate $(2\pi\xi[N])^{-2}$, in largely dashed green.

IV. BOUNDS FROM PRIMORDIAL BLACK HOLES

Now let us try to compare this with the existing bounds on the power spectrum coming from the non-detection of primordial black holes. These will form if at horizon re-entry (i.e. smoothing ζ on scales of order H) we have $\zeta > \zeta_c$, with $\zeta_c \sim 1$ denoting the critical value leading to black hole formation. If one assumes that ζ follows a Gaussian distribution (with $\langle \zeta \rangle = 0$) one can express the probability of having $\zeta > \zeta_c$ in terms of the variance $\langle \zeta^2 \rangle$ by analyzing the Gaussian probability distribution function. This probability corresponds to the fraction of space β that can collapse to form horizon-sized black holes. Hawking evaporation and present day gravitational effects constrain this fraction β . Typically one finds β in the range $(10^{-28} - 10^{-5})$, with the strongest bounds coming from CMB anisotropies [37] (spectral dis-

tortion and photodissociation of deuterium lead to a bound $\beta \lesssim 10^{-20}$, as for example in [36]). Setting $\zeta_c = 1$ gives for the upper bound on the power spectrum [40]

$$\Delta_{\zeta,c}^2(k) \simeq \langle \zeta(x)^2 \rangle \simeq 0.008 - 0.05. \quad (34)$$

Here the lower bound corresponds to $\beta = 10^{-28}$ and the upper bound to $\beta = 10^{-5}$.

However, in our case ζ does not follow a Gaussian distribution. Instead we have (see Appendix (B))

$$\zeta = -\frac{\alpha(\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle)}{3\beta H \dot{\chi}}. \quad (35)$$

The stochastic properties of the vector field δA are close to those in a free theory, i.e. it has Gaussian perturbations around $\langle \delta A \rangle = 0$. As a consequence we can write⁶

$$\zeta = g^2 - \langle g^2 \rangle \quad (36)$$

with g a Gaussian distributed field. This model was studied in [40] and we follow that derivation (see also [38, 39]). The probability distribution function of ζ follows from setting $P(\zeta)d\zeta = P(g)dg$, and takes the form

$$P(\zeta) = \frac{1}{\sqrt{2\pi(\zeta + \sigma^2)}\sigma} e^{-\frac{\zeta + \sigma^2}{2\sigma^2}}, \quad (37)$$

with $\sigma^2 \equiv \langle g^2 \rangle$. For a given value of β we can again infer σ^2 . Setting $t \equiv \frac{\zeta_c}{\sigma^2} + 1$ (and $t_c \equiv \frac{\zeta_c}{\sigma^2} + 1$) we have $d\zeta = \sigma^2 dt$ which gives

$$\beta = \int_{\zeta_c}^{\infty} P(\zeta)d\zeta = \int_{t_c}^{\infty} \frac{e^{-\frac{t}{2}}}{\sqrt{2\pi t}} dt = \text{Erfc}\left(\sqrt{\frac{t_c}{2}}\right), \quad (38)$$

where $\text{Erfc}(x) \equiv 1 - \text{Erf}(x)$ is the complementary error function. Taking again β in the range $10^{-28} - 10^{-5}$ one gets a tighter upper bound on the power spectrum than in the Gaussian case:

$$\begin{aligned} \Delta_{\zeta,c}^2(k) &\simeq \langle \zeta(x)^2 \rangle = 2\langle g^2 \rangle^2 \\ &\simeq 1.3 \cdot 10^{-4} - 5.8 \cdot 10^{-3}. \end{aligned} \quad (39)$$

Now let us estimate what value of β is relevant for our investigation.

At the end of inflation, the total mass concentrated in the volume associated with perturbations leaving the horizon N e-foldings before the end of inflation with the Hubble constant H can be estimated by

$$M_N \simeq \frac{4}{3}\pi\rho r^3 \simeq \frac{4\pi M_p^2}{H} e^{3N}, \quad (40)$$

where we reinserted the reduced Planck mass M_p , which was set to one in the rest of the paper, and H is calculated at the end of inflation. In order to study the subsequent evolution of matter in the comoving volume corresponding to this part of the universe, one should distinguish between two specific possibilities depending on the dynamics of reheating after inflation, discussed in section VIII.

If reheating is not very efficient, then the universe for a long time remains dominated by scalar field oscillations, with the average equation of state $p = 0$. In this case, the total mass in the comoving volume does not change, and therefore at the moment when the black hole forms, its mass M_{BH} is equal to M_N evaluated in (40). For the parameters of our model, this gives an estimate (see appendix E for details)

$$M_{BH} \simeq 10 e^{3N} \text{ g}. \quad (41)$$

On the other hand, if reheating is efficient, then the post-inflationary universe is populated by ultra relativistic particles and the energy density in comoving volume scales inversely proportional to the expansion of the universe. In this case, the black hole mass can be estimated as (see appendix E)

$$M_{BH} \simeq 10 e^{2N} \text{ g}. \quad (42)$$

In our estimates of the black hole production we will assume the latter possibility, though in general one may have a sequence of the first and the second regime. The final conclusion will only mildly depend on the choice between these two possibilities.

Now, the bounds on β in terms of the would-be black hole mass M_{BH} were given in [36] and updated in [37]. Here we follow the result in [37].⁷ Using (38) and our estimates of the black hole mass as a function of N , we can translate this into a bound on the power spectrum as a function of N . The result is in Figure 5.

Our estimate (33) *violates this bound* for all $N \lesssim 20$ by a factor of about six. Since we have made some approximations both in deriving the late time power spectrum

⁶ Here we can safely neglect the linear term, which is just the standard vacuum slow-roll contribution to ζ . See also our estimate for f_{NL} at small scales in Appendix (D).

⁷ However, we do not take the constraints for $M_{BH} < 10^8$ g into account, as these are either very model dependent, or assume that black hole evaporation leaves stable relics.

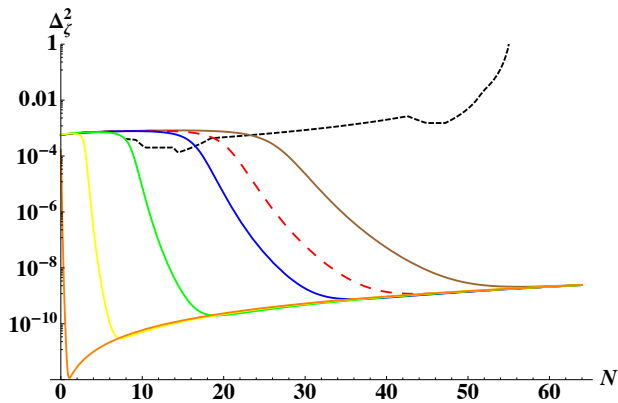


FIG. 5. Evolution of our estimate for the power spectrum as a function of N . In dashed red is the result for $\xi[N=64] = 2.2$. Other lines are for $\xi[N=64] = 2.5$ (solid brown), $\xi[N=64] = 2$ (solid blue), $\xi[N=64] = 1.5$ (solid green), $\xi[N=64] = 1$ (solid yellow) and $\xi[N=64] = 0.5$ (solid orange). The black hole bound is in dashed black.

and in deriving its observational upper bound, our estimate could well be off by some order one factor and therefore we can not draw a definitive conclusion. It is clear though that the parameter values giving rise to an observable but not yet ruled out violation of scale invariance and non-Gaussianity in the CMB-window produce a late time power spectrum that comes at least very close to the primordial black hole bound. A more precise computation is needed to establish whether this bound is actually violated or not.

However, if such a computation revealed that primordial black holes do indeed constrain these models, that would yield a much stronger bound on ξ as the ones coming from non-Gaussianity and the violation of scale invariance. Since we have seen that the power spectrum has a late-time asymptotic of $(2\pi\xi[N])^{-1}$, this problem persists on a wide range of values for ξ .

For all values of ξ , our estimate for the power spectrum sharply increases before the end of inflation, the closer to the end the smaller ξ is. However, if we disregard black hole bounds for $M_{BH} \lesssim 10^8$ g, which rely on uncertain model dependent assumptions, there are no black hole bounds for $N \lesssim 8$. From figure 5 we then see that we get

$$\xi(N_{CMB}) \lesssim 1.5 \quad (43)$$

for the bound on ξ at CMB scales from primordial black hole production. In terms of the coupling constant α , this bound implies the constraint

$$\alpha \lesssim 23. \quad (44)$$

This bound is derived using (42), i.e. radiation domination right after the end of inflation. This assumption fixes

the expansion history of the universe and therefore specifies $N_{CMB} \simeq 64$, for the N corresponding to CMB scales (see appendix E for a derivation). This is required for consistency but changes the numerics very little. Therefore in all other sections we still use $N_{CMB} = 60$.

For the matter domination regime, the black hole masses would be greater, for a given N , see (41), and therefore we would have a slightly stronger constraint on ξ and α . We find $\xi \lesssim 1.3$ which corresponds to $\alpha \lesssim 20$. Instead of concentrating on it, we will now investigate the model where non-Gaussian perturbations may be generated for much smaller ξ and α , without leading to the primordial black hole problem.

V. LOCAL NON-GAUSSIANITY FROM HEAVY VECTOR FIELDS

Now let us turn to a scenario, described in [33], in which the produced gauge fields are massive. The production of gauge quanta decreases with the mass of the gauge fields: for $m_A \sim \xi H$ all production is killed. In this scenario, the gauge fields get their mass via the Higgs mechanism. Fluctuations in the Higgs field h lead to fluctuations in m_A , which in turn generate fluctuations in the amount of produced gauge quanta, and therefore in the amount of extra friction in the dynamics of χ and H . In the end, one has perturbations in ΔN , namely the number of extra e-folds of inflation due to gauge field production. This leads to a non-Gaussian signal in the CMB of the local type [33]. Using the δN formalism one finds

$$f_{NL}^{\text{local}} \sim 10^2 \left(\frac{\Delta N_{\text{max}}^{3/4} e}{\xi 10^{-3}} \right)^4 \left(\frac{m_A}{\xi H} \right)^2. \quad (45)$$

Here h is the Higgs-like field responsible for the spontaneous symmetry breaking that gives a mass to the gauge fields, e is its $U(1)$ charge, $m_A = eh$ and assumed a quadratic potential $H = \frac{m\chi}{\sqrt{6}}$.

For a complete description we refer the reader to the original reference [33], section 7. Here we only want to stress that this scenario can also work for $\xi \sim 1$. Then it will surely satisfy the bounds from primordial black holes. However, this scenario requires that $m_h \ll H$, which seems rather unnatural. In the next two sections we want to explain how this model, that can produce an observable local non-Gaussian signal, can be embedded in a supergravity model that is a slight extension of the models considered so far. In this model one can easily have $m_h \ll H$, as we will argue.

VI. CURVATON SCENARIO AND CHAOTIC INFLATION IN SUPERGRAVITY

We go back to the supergravity scenario described in section I, that we want to extend by making the vector fields heavy because of the Higgs effect. We will start with the simplest version of chaotic inflation proposed in [4], with the superpotential

$$W = mS\Phi, \quad (46)$$

and Kähler potential

$$\mathcal{K} = S\bar{S} - \frac{1}{2}(\Phi + \bar{\Phi})^2. \quad (47)$$

The Kähler potential does not depend on the phase of the field S and on the imaginary part of the field Φ . It is convenient to represent the fields S and Φ as $S = \sigma e^{i\theta}/\sqrt{2}$ and $\Phi = (\phi + i\chi)/\sqrt{2}$. The field χ plays the role of the inflaton field, with the quadratic potential, as in the simplest version of the chaotic inflation scenario [46]:

$$V(\chi) = \frac{m^2}{2}\chi^2, \quad (48)$$

Near the inflationary trajectory with $S = 0$, the mass squared of the real part of the field Φ is $m_\phi^2 = 6H^2 + m^2$, where $H^2 = m^2\chi^2/6$. During inflation $m_\phi^2 > 6H^2$. Therefore the field ϕ quickly rolls towards its equilibrium position at $\phi = 0$, and perturbations of this field are not generated. Meanwhile the field S is light, its mass is well below H , and can play the role of the Higgs field discussed in the previous section. Hence we assume that S has some $U(1)$ charge e and a minimal coupling to gauge fields $|(\partial_\mu + ieA_\mu)S|^2$. This is the main new ingredient that we introduce now to our model.

Both components of the field S may remain light during inflation, and therefore inflationary perturbations of these fields can be generated [11]. The fluctuations of the field σ are generated just as the fluctuations of the curvaton field [25, 26], though in the present context its fluctuations will be important for a different reason. They will be responsible for the fluctuations of the mass of the vector field, in accordance with the scenario described in the previous section.

The potential of the fields χ , σ at $\phi = 0$ is

$$V(\chi, \sigma) = \frac{m^2}{2}e^{\sigma^2/2} \left[\chi^2 + \sigma^2 + \frac{\chi^2}{4}\sigma^2(\sigma^2 - 2) \right]. \quad (49)$$

For $\sigma \ll 1$ one has

$$V(\chi, \sigma) = \frac{m^2\chi^2}{2} + \frac{m^2\sigma^2}{2} + \frac{m^2\chi^2\sigma^4}{16}. \quad (50)$$

The effective mass squared of the field σ in this regime is

$$m_\sigma^2 = m^2 + \frac{3}{4}m^2\chi^2\sigma^2. \quad (51)$$

Note that $m_\sigma^2 = m^2$ for $\chi\sigma \ll 1$. During inflation $m^2 \ll H^2$, and therefore inflationary perturbations of the field σ can be generated. At $\chi\sigma \gtrsim 1$, the effective mass squared of the field σ is dominated by the term $\frac{3}{4}m^2\chi^2\sigma^2 = \frac{9}{2}H^2\sigma^2 > m^2$. For $\sigma \ll 1$ one still has $m_\sigma^2 \ll H^2$, so the perturbations of the field σ are generated in this regime as well. At $\sigma \gtrsim 1$ the potential becomes exponentially steep, $m_\sigma^2 \gg H^2$. Therefore inflationary fluctuations of this field are generated only for $\sigma \lesssim 1$. The steepness of the curvaton potential at $\sigma \gtrsim 1$ protects us from extremely large perturbations of the curvaton field which otherwise could be produced during eternal inflation [47, 48].

If one does not take into account the curvaton fluctuations in this scenario and study only the usual inflaton fluctuations [49], then the COBE normalization requires $m \sim 6.35 \times 10^{-6}$, in the system of units $M_p = 1$. With an account taken of vector field production modulated by curvaton fluctuations, inflation lasts a bit longer, and therefore one has to take the inflaton mass a bit larger. For $\xi = 1$ one needs $m \sim 6.45 \times 10^{-6}$.

VII. STOCHASTIC APPROACH

We will begin our study with investigation of the behavior of the distribution of the fluctuations of the curvaton field σ , following [26]. During inflation, the long-wavelength distribution of this field generated at the early stages of inflation behaves as a nearly homogeneous classical field, which satisfies the equation

$$3H\dot{\sigma} + V_\sigma = 0, \quad (52)$$

which can be also written as

$$\frac{d\sigma^2}{dt} = -\frac{2V_\sigma\sigma}{3H}. \quad (53)$$

However, each time interval H^{-1} new fluctuations of the scalar field are generated, with an average amplitude squared⁸

$$\langle \delta\sigma^2 \rangle = \frac{H^2}{2\pi^2}. \quad (54)$$

⁸ For a real massless field we would get $\langle \delta\sigma^2 \rangle = \frac{H^2}{4\pi^2}$. An extra coefficient 2 appears in (54) because the field S is complex, so its absolute value changes faster because of independent fluctuations of its two components. One could argue that in the unitary gauge we only have one scalar degree of freedom. However, unitary gauge is problematic in the description of the Brownian motion and cosmic string formation in the Higgs model. We present the results which should be valid in the regime of small gauge coupling constant e . Our main conclusions are unaffected by this factor of 2 issue.

The wavelength of these fluctuations is rapidly stretched by inflation. This effect increases the average value of the squared of the classical field σ in a process similar to the Brownian motion. As a result, the square of the field σ at any given point with an account taken of inflationary fluctuations changes, in average, with the speed which differs from the predictions of the classical equation of motion by $\frac{H^3}{4\pi^2}$:

$$\frac{d\sigma^2}{dt} = -\frac{2V_\sigma \sigma}{3H} + \frac{H^3}{2\pi^2}. \quad (55)$$

Using $3H\dot{\chi} = -V_\chi$, one can rewrite this equation as

$$\frac{d\sigma^2}{d\chi} = \frac{2V_\sigma \sigma}{V_\chi} - \frac{V^2}{6\pi^2 V_\chi}. \quad (56)$$

By solving this equation with different boundary conditions, one can find the most probable value of the locally homogeneous field σ . In the supergravity model that we study, this equation has an important advantage [26]: For a very broad choice of initial conditions, its solutions rapidly converge to an attractor solution

$$\sigma(\chi) \approx \frac{m^{2/3} \chi}{2\sqrt{3}\pi^{2/3}} \left(\frac{\Gamma(1/3)}{\Gamma(2/3)} \right)^{1/2} \approx 0.19 m^{2/3} \chi. \quad (57)$$

If the amplitude of perturbations of metric is determined mainly by the inflaton perturbations, one should have m in the range of 6.35×10^{-6} to 7.2×10^{-6} , depending on ξ (these two values correspond to $\xi = 0$ and $\xi[N = 60] = 2.2$, respectively.)

Now we will use the expression for $f_{\text{NL}}^{\text{local}}$ (45) obtained in [33], for $m_A = e\sigma$ and $H = \frac{m\chi}{\sqrt{6}}$, under condition $m_A \lesssim \xi H$:

$$f_{\text{NL}}^{\text{local}} \sim 2.15 \times 10^{13} \Delta N_{\text{max}}^3 e^6 \xi^{-6} m^{-2/3}. \quad (58)$$

As we see, this expression is very sensitive to the value of the gauge coupling e . For any value of ξ and m , one can calculate ΔN_{max} , and then one can find a particular value of the gauge coupling e which results in the perturbations with a desirable value of $f_{\text{NL}}^{\text{local}}$:

$$e \simeq 6 \times 10^{-3} \frac{\xi m^{1/9}}{\Delta N_{\text{max}}^{1/2}} (f_{\text{NL}}^{\text{local}})^{1/6}. \quad (59)$$

One may consider some numerical examples. For $\xi \sim 1$ one finds $\Delta N_{\text{max}} \sim 1$ (the example considered in [33]). Then for $m \sim 6.5 \times 10^{-6}$ one may have, for example, $f_{\text{NL}}^{\text{local}} \sim 30$ for $e \sim 2.8 \times 10^{-3}$. For $\xi \sim 0.5$ one finds $\Delta N_{\text{max}} \sim 0.044$, so one may have $f_{\text{NL}}^{\text{local}} \sim 30$ for $e \sim 6.7 \times 10^{-3}$. Thus the required gauge couplings are pretty small. They could correspond, e.g., to gauge interactions with vector fields in the hidden sector.

Any further decrease of ξ well below 0.5 rapidly shuts down generation of the vector field fluctuations and makes ΔN_{max} exponentially small, which does not allow production of large non-Gaussianity. One could hope, following (58), that it is possible to compensate any decrease of ξ by an increase of e , but one cannot increase e too much without violating the condition $m_A \lesssim \xi H$ required for the vector field production.

Until now, we assumed that the amplitude of the metric perturbations is dominated by the usual inflaton perturbations. However, one may consider models with $m \ll 6 \times 10^{-6}$ and study a possibility that the leading contribution to perturbations of metric is given by the curvaton-related perturbations. As an example, we will assume that $m \sim 10^{-9}$, in which case inflaton perturbations give a negligibly small contribution to the COBE normalized amplitude of perturbations of metric. Using (7.23) of [33] and (57) one finds that in this case COBE normalization for $\xi = 1$ requires $e \approx 8 \times 10^{-4}$, which yields $f_{\text{NL}} \sim 7$, which is almost too small to be observed. By lowering ξ , one can exponentially decrease ΔN_{max} and get non-Gaussianity in the observable range for somewhat larger values of the gauge coupling.

Now let us add a bit of black magic. When we calculated $\sigma(\chi)$ in (57), in fact we calculated the typical deviation of the field σ from 0, or, in other words, the width of its distribution. The field σ can be twice as large as our result indicates, even though the probability of that would be somewhat suppressed. More importantly, it can be much smaller than the value which we calculated; the probability of this to happen is nearly flatly distributed in the interval from 0 to $\sigma(\chi)$.

Notice that the amplitude of perturbations of the metric is proportional to $e^2 \sigma$, which we should keep constant to maintain the COBE normalization. Meanwhile f_{NL} is proportional to $e^6 \sigma^2 = \frac{(e^2 \sigma)^3}{\sigma}$. Thus, for the given amplitude of perturbations of metric, $f_{\text{NL}} \sim 1/\sigma$, so one can get f_{NL} which is two times greater than our estimate in those parts of the universe where $\sigma = \sigma(\chi)/2$. As we just argued, parts of the universe with $\sigma \lesssim \sigma(\chi)$ are quite abundant. Thus we can easily have a proper amplitude of perturbations and f_{NL} in the range of $\mathcal{O}(10)$ or higher in this scenario.

One can make a similar trick in the previously studied regime where the amplitude of perturbations of metric was dominated by the inflaton contribution. In that case, this amplitude does not depend on σ , whereas f_{NL} is proportional to σ^2 . This means that one can live, with a significant probability, in the areas where f_{NL} is much smaller than the result obtained above.

Thus, just like in the general curvaton scenario, we are dealing with the ‘‘curvaton web’’, where the universe becomes divided into different exponentially large parts with different values of the amplitude of perturbations of

metric and different values of f_{NL} [26, 47].

VIII. GAUGE FIELD PRODUCTION IN SUGRA INFLATION: REHEATING

We have found that the coupling $\chi F\tilde{F}$ needed for reheating in (the pseudo scalar variant of) the new class of SUGRA inflation models proposed in [1–3] can as well yield an observable non-Gaussian signal. It only remains to be seen what the effects of the typically needed values for ξ are for the reheating in the combined model.

In [3] the reheating temperature T_R for the decay of a scalar inflaton field to two photons due to the interaction $\frac{\alpha}{4}\phi F^2$ was estimated as

$$T_R \approx \sqrt{2}\alpha \times 10^9 \text{ GeV}. \quad (60)$$

A similar estimate is valid in our case. One may also represent it in an equivalent way using relation $\frac{\alpha}{4} = -\frac{\xi H}{2\chi}$, and an expression for the slow-roll parameter $\epsilon = \frac{\dot{\chi}^2}{2H^2}$

$$T_R \approx \frac{2\xi}{\sqrt{\epsilon}} \times 10^9 \text{ GeV}. \quad (61)$$

As long as one can describe reheating as a particle by particle decay, reheating in inflationary models of this type does not depend much on whether the inflaton field is a scalar or a pseudo scalar. In both types of models, one may consider interactions with $\alpha \ll 1$, which results in reheating temperature $T_R \lesssim 10^8 \text{ GeV}$. This solves the cosmological gravitino problem for gravitino in the typical mass range $m_{3/2} \lesssim 1 \text{ TeV}$.

However, for $\alpha \gtrsim 1$, which is required for production of non-Gaussianity in the models based on the pseudo scalar inflaton, an estimate described above gives $T_R > 10^9 \text{ GeV}$. It is good for the theory of leptogenesis, but it could be bad from the point of view of the gravitino problem. Moreover, for $\alpha \gtrsim 1$ an entirely different mechanism of reheating is operating. At the end of inflation, when the time-dependent parameter ξ grows and becomes large, a significant fraction of the energy of the inflaton field gradually becomes converted to the energy of the vector field (see Figure 6). This is a very efficient mechanism, which may lead to a very rapid thermalization of energy in the hidden sector. This may exacerbate the gravitino problem. Fortunately, this problem does not appear for superheavy gravitino with mass $m_{3/2} \gtrsim 10^2 \text{ TeV}$. Such gravitinos appear in many versions of the models of mini-split supersymmetry, which became quite popular during the last few years, see [50] and references therein.

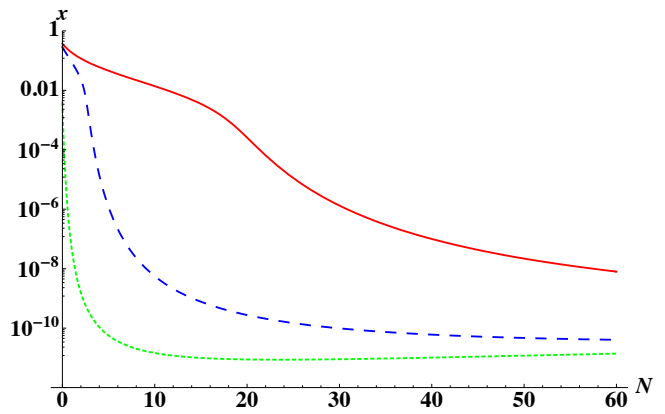


FIG. 6. Evolution of the normalized energy of the vector field, $x \equiv \frac{1}{2}(E^2 + B^2)/3H^2$ as a function of N , for $\xi[N=60] = 2.2$ (solid red), $\xi[N=60] = 1.0$ (largely dashed blue) and $\xi[N=60] = 0.5$ (finely dashed green).

IX. CONCLUSIONS

The new class of chaotic inflation models in supergravity needs a gauge-gauge-inflaton coupling for reheating. The inclusion of this coupling can produce gauge fields and can provide a Planck-observable but not yet ruled out non-Gaussian signal in the CMB.

In this article we have studied two possible realizations of this scenario. Taking the parameter $\xi \simeq 2.2 - 2.5$ ($\alpha \simeq 32 - 37$) produces a large amount of gauge quanta, that by inverse decay give rise to an equilateral non-Gaussianity in the CMB, as studied in [30, 31]. However, we have estimated that towards the end of inflation the power spectrum grows so much that the model may be ruled out because it overproduces primordial black holes. As our order-one estimate lands within a factor of six from the critical black hole bound on the power spectrum (with the non-Gaussian nature of the signal taken into account), we need a more precise computation to draw a definitive conclusion.

In the second scenario, where the produced gauge fields are massive due to the Higgs effect in presence of a light curvaton-type field, one can take a smaller value for ξ , of order 0.5 - 1, corresponding to α from 8 to 15. Then the model is free of black hole trouble. In this case, fluctuations in the curvaton field modulate the duration of inflation and can give rise to adiabatic non-Gaussian perturbations of the local type with $f_{\text{NL}} \sim \mathcal{O}(10 - 100)$. For smaller values of α , we return to the standard chaotic inflation scenario with Gaussian adiabatic perturbations.

ACKNOWLEDGMENTS

A.L. is supported by NSF grant PHY-0756174. E. P. is supported in part by the Department of Energy grant DE-FG02-91ER-40671. S. M. wants to thank the Stanford Institute for Theoretical Physics for great hospitality during his two month's visit in which this work was initiated.

Appendix A: Variance of $\vec{E} \cdot \vec{B}$

The variance of $\vec{E} \cdot \vec{B}$ is defined as

$$\sigma^2 \equiv \langle (\vec{E} \cdot \vec{B})^2 \rangle - \langle \vec{E} \cdot \vec{B} \rangle^2 \quad (\text{A1})$$

$$= \langle E_i E_j \rangle \langle B_i B_j \rangle + \langle E_i B_j \rangle \langle B_i E_j \rangle. \quad (\text{A2})$$

We find

$$\begin{aligned} \langle E_i E_j \rangle \langle B_i B_j \rangle &= \frac{1}{a^8} \int \frac{dk dq}{(2\pi)^6} |A'(k)|^2 |A(q)|^2 q^4 k^2 \\ &\quad \int d^2 \Omega_k d^2 \Omega_q \epsilon_i(k) \epsilon_j(q) \epsilon_j^*(k) \epsilon_i^*(q), \\ \langle E_i B_j \rangle \langle B_i E_j \rangle &= \frac{1}{a^8} \int \frac{dk dq}{(2\pi)^6} A(k) A'^*(k) A'(q) A^*(q) q^3 k^3 \\ &\quad \int d^2 \Omega_k d^2 \Omega_q \epsilon_i(k) \epsilon_j(q) \epsilon_j^*(k) \epsilon_i^*(q). \end{aligned} \quad (\text{A3})$$

The angular integral gives $(4\pi)^2/3$, i.e. a third of the whole sphere. The integrals over the modulus are similar to the one in [31] and are computed in the same way

$$I_2 = \frac{1}{a^4} \int \frac{dk}{(2\pi)^3} |A'(k)|^2 k^2 \simeq 2.2 \cdot 10^{-5} \frac{H^4}{\xi^3} e^{2\pi\xi}, \quad (\text{A4})$$

$$I_3 = \frac{1}{a^4} \int \frac{dk}{(2\pi)^3} |A'(k)|^2 k^2 \simeq 1.9 \cdot 10^{-5} \frac{H^4}{\xi^4} e^{2\pi\xi}, \quad (\text{A5})$$

$$I_4 = \frac{1}{a^4} \int \frac{dk}{(2\pi)^3} |A(k)|^2 k^4 \simeq 1.9 \cdot 10^{-5} \frac{H^4}{\xi^5} e^{2\pi\xi}. \quad (\text{A6})$$

Putting things together one finds

$$\sigma = \sqrt{\frac{(4\pi)^2}{3} (I_3^2 + I_2 I_4)} \quad (\text{A7})$$

$$= 2.0 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi} \simeq -\langle \vec{E} \cdot \vec{B} \rangle. \quad (\text{A8})$$

Appendix B: Power spectrum estimate

In [30, 31] the power spectrum (24) has been obtained by the Green's function method. In [32] a quick estimate was introduced to compute the power spectrum in the case of large backreaction ($\beta \gg 1$). Here we want to review and further explore this estimate, showing how it leads to (31) and also how, in the case of negligible backreaction, it approximates the precise result (24) within a factor of two.

The full equation of motion for the perturbation $\delta\chi$ is (in real space)

$$\delta\ddot{\chi} + 3\beta H \delta\dot{\chi} - \frac{\nabla^2}{a^2} \delta\chi + m^2 \delta\chi = \alpha \left[\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle \right], \quad (\text{B1})$$

with

$$\beta \equiv 1 - 2\pi\xi\alpha \frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}}. \quad (\text{B2})$$

Near horizon crossing we can estimate $\partial \sim H$. Since we have, near horizon crossing, $H^2 = \frac{k^2}{a^2}$, the first term cancels the third one. The second term can be approximated as $3\beta H^2 \delta\chi$. The last term on the left hand side is just a slow-roll correction and can be discarded. This directly gives

$$\delta\chi \approx \frac{\alpha \left(\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle \right)}{3\beta H^2} \quad (\text{B3})$$

and therefore we have

$$\zeta \equiv -\frac{H}{\dot{\chi}} \delta\chi \approx -\frac{\alpha \left(\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle \right)}{3\beta H \dot{\chi}}. \quad (\text{B4})$$

For the position space two point function of ζ we immediately get

$$\begin{aligned} \langle \zeta(x)^2 \rangle &\equiv \frac{H^2}{\dot{\chi}^2} \langle \delta\chi^2 \rangle \approx \frac{H^2}{\dot{\chi}^2} \left(\frac{\alpha\sigma}{3\beta H^2} \right)^2 \\ &= \left(\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3\beta H \dot{\chi}} \right)^2 \end{aligned} \quad (\text{B5})$$

with σ the variance computed in the previous subsection.

To compare the position space power spectrum with

the momentum space power spectrum we use

$$\begin{aligned} \langle \zeta(\vec{k}) \zeta(\vec{k}') \rangle &\equiv (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P(k), \quad P(k) \equiv \frac{2\pi^2 \Delta_\zeta^2(k)}{k^3}, \\ \langle \zeta(x)^2 \rangle &= \int d\ln k \Delta_\zeta^2(k) \simeq \mathcal{O}(1) \Delta_\zeta^2(k). \end{aligned} \quad (\text{B6})$$

This gives the result (31):

$$\Delta_\zeta^2(k) \simeq \langle \zeta(x)^2 \rangle = \left(\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3\beta H \dot{\chi}} \right)^2. \quad (\text{B7})$$

This expression has been plotted in figure 4.

Now when backreaction is strong we can approximate $\beta \approx -2\pi\xi\alpha \frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}}$, which immediately gives the approximation (32)

$$\Delta_\zeta^2(k) = \frac{1}{(2\pi\xi)^2}. \quad (\text{B8})$$

We can as well make an approximation for the case where $\beta \approx 1$ (negligible backreaction) and compare the result with the precise result (24), just to see how well this whole approximation works. For $\beta = 1$ we have

$$\Delta_\zeta^2(k) = \left(\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}} \right)^2. \quad (\text{B9})$$

Upon using the estimate for $\langle \vec{E} \cdot \vec{B} \rangle$ found in [30, 31]

$$\langle \vec{E} \cdot \vec{B} \rangle \approx 2.4 \times 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi} \quad (\text{B10})$$

and

$$\alpha \equiv -\frac{2H\xi}{\dot{\chi}} \quad (\text{B11})$$

we find

$$\begin{aligned} \Delta_\zeta^2(k) &= \frac{4H^2\xi^2}{\dot{\chi}^2} \times 5.76 \times 10^{-8} \times \frac{H^8}{\xi^8} e^{4\pi\xi} \times \frac{1}{9H^2\dot{\chi}^2} \\ &= 2.56 \times 10^{-8} \times \frac{H^8}{\dot{\chi}^4} \times \frac{e^{4\pi\xi}}{\xi^6} \\ &= 2.56 \times 10^{-8} \times \left(\frac{H^2}{2\pi\dot{\chi}} \right)^4 \times (2\pi)^4 \times \frac{e^{4\pi\xi}}{\xi^6} \\ &= 4.0 \times 10^{-5} \times \Delta_{\zeta,\text{sr}}^4(k) \times \frac{e^{4\pi\xi}}{\xi^6}. \end{aligned} \quad (\text{B12})$$

This can be compared with the more precise result computed in [30, 31] that uses the Green's function approach

$$\Delta_\zeta^2(k) = \Delta_{\zeta,\text{sr}}^4(k) \times f_2(\xi) \times e^{4\pi\xi} \quad (\text{B13})$$

$$\simeq \Delta_{\zeta,\text{sr}}^4(k) \frac{7.5 \times 10^{-5}}{\xi^6} \times e^{4\pi\xi}. \quad (\text{B14})$$

where in the second line we used the large ξ limit for f_2 . We infer that this quick estimate is off by a factor less than two.

Actually, for some ξ the estimate comes even closer than this ratio $\frac{7.5}{4}$. Let us examine the situation at $\xi = 3$ (which, for $\xi(N = 60) = 2.2$), corresponds to $N \approx 35$). Above, we approximated the numerical function $f_2(\xi)$ by $\frac{7.5 \times 10^{-5}}{\xi^6}$ which yields an overestimate by a factor of 1.3. At the other hand, we also approximated the numerically found result for $\langle \vec{E} \cdot \vec{B} \rangle$ by the estimate B10, which is an underestimate, that for $\xi = 3$ only captures a fraction of 0.73 of the true $\langle \vec{E} \cdot \vec{B} \rangle$. Putting everything together one finds that, at $\xi = 3$ ($N = 35$), our estimate B7 with β set to one overestimates the precisely computed numerical result (B13) by a factor of

$$\frac{4}{7.5} \times \frac{1.3}{(0.73)^2} \approx 1.3. \quad (\text{B15})$$

At $\xi=2.2$ ($N=60$) we find that our estimate B7 overestimates the precisely computed result by a factor of 2.5.

Now one might introduce a fudge factor such that at some preferred value for ξ our approximation precisely matches the numerically computed result. However, we have just seen that the inclusion of such a fudge factor will induce only a small shift in our estimate that we anyway only trust up to corrections of order one. Besides, the fudge factor would always be arbitrary, as it depends on the preferred value of ξ where it makes both signals match. Therefore it seems safe to neglect it altogether. In Figure 7 we have for once plotted how the total power spectrum (including the standard slow-roll contribution) would shift from such a correction. In the rest of the paper we work with our uncorrected estimate for the power spectrum.

N.B. This estimate involves only the gauge field contribution to the power spectrum. Apart from that there is always the standard slow-roll component $\Delta_{\zeta,\text{sr}}^2(k)$. This is the dominant contribution on CMB scales. That is why any estimate of the total power spectrum matches the precise result so well on CMB scales, whatever order one fudge factor one chooses.

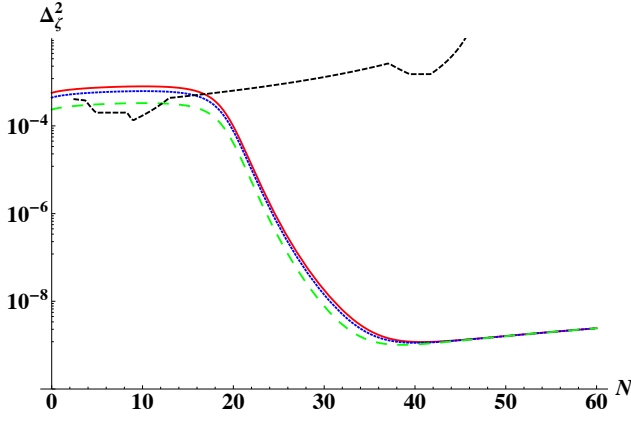


FIG. 7. Evolution of the power spectrum as in Figure 4, still for $\xi[N=60] = 2.2$. The red solid line is our estimate. The blue, thinly dashed line is our estimate corrected with a fudge factor of 1.3. The green, largely dashed line is our estimate corrected with a fudge factor of 2.5. All signals remain within an order one factor from the black hole bounds in dashed black.

Appendix C: Skewness of $\vec{E} \cdot \vec{B}$

We want to compute

$$\begin{aligned} \tau^3 &\equiv \langle (\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle)^3 \rangle \\ &= \langle (\vec{E} \cdot \vec{B})^3 \rangle - 4\langle \vec{E} \cdot \vec{B} \rangle^3 \\ &\simeq \langle (\vec{E} \cdot \vec{B})^3 \rangle_c + 3\langle \vec{E} \cdot \vec{B} \rangle^3, \end{aligned} \quad (C1)$$

where we used that $\langle (\vec{E} \cdot \vec{B})^2 \rangle \simeq 2\langle \vec{E} \cdot \vec{B} \rangle^2$ from the previous section and in the last step we recognized that there are $1 + 3 \times 2 = 7$ non-connected diagrams in $\langle (\vec{E} \cdot \vec{B})^3 \rangle$, each one equals to $\langle \vec{E} \cdot \vec{B} \rangle^3$. Using Wick's theorem we find many terms. All of them have the same angular integral

$$\begin{aligned} \int d^2\Omega_{k_1} d^2\Omega_{k_2} d^2\Omega_{k_3} \epsilon_i(k_1) \epsilon_i(k_2) \epsilon_j^*(k_1) \epsilon_j(k_3) \epsilon_j^*(k_2) \epsilon_j^*(k_3) \\ = \frac{2\pi^5}{3}. \end{aligned} \quad (C2)$$

Counting all the possible pairwise contractions one finds

$$\begin{aligned} \langle (\vec{E} \cdot \vec{B})^3 \rangle_c &= \frac{2\pi^5}{3} (2I_3^3 + I_2 I_3 I_4) \\ &= \left[2.4 \cdot 10^{-4} \frac{H^4}{\xi^4} e^{2\pi\xi} \right]^3 \simeq -\langle \vec{E} \cdot \vec{B} \rangle^3 \end{aligned} \quad (C3)$$

and therefore

$$\tau^3 \simeq 2\langle \vec{E} \cdot \vec{B} \rangle^3. \quad (C4)$$

Appendix D: Bispectrum and f_{NL} estimate

The position space three point function of ζ can be directly generalized from (B5):

$$\begin{aligned} \langle \zeta(x)^3 \rangle &\equiv -\frac{H^3}{\dot{\chi}^3} \langle \delta\chi^3 \rangle \approx -\frac{H^3}{\dot{\chi}^3} \left(\frac{\alpha\tau}{3\beta H^2} \right)^3 \\ &= -2 \left(\frac{\alpha \langle \vec{E} \cdot \vec{B} \rangle}{3\beta H \dot{\chi}} \right)^3, \end{aligned} \quad (D1)$$

where we used the definition of the skewness τ^3 (C1) and its estimate (C4). $\langle \zeta(x)^3 \rangle$ is positive. (Again: we work with negative $\dot{\chi}$ which yields positive $\langle \vec{E} \cdot \vec{B} \rangle$, while working with $\dot{\chi} > 0$ would give $\langle \vec{E} \cdot \vec{B} \rangle < 0$.)

Let us first analyze this result in the regime where backreaction is negligible, i.e. $\beta = 1$. Using (B10) and (B11) we get

$$\begin{aligned} \langle \zeta(\vec{x})^3 \rangle &\simeq 2 \frac{8}{27} (2.4 \times 10^{-4})^3 \frac{H^{12} e^{6\pi\xi}}{\xi^9 \dot{\chi}^6} \\ &\simeq 8.2 \times 10^{-12} \frac{H^{12} e^{6\pi\xi}}{\xi^9 \dot{\chi}^6}. \end{aligned} \quad (D2)$$

Now we want to compare this with the momentum space bispectrum $B(k)$, defined via

$$\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle \equiv (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \quad (D3)$$

for which we can write

$$\langle \zeta(\vec{x})^3 \rangle = \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} B(\vec{k}_1, \vec{k}_2, -\vec{k}_1 - \vec{k}_2). \quad (D4)$$

When non-Gaussianity is large mostly on equilateral triangles, the integral is supported in the region $k_2 \simeq k_1$ and $\theta_{12} \simeq \pi/3$. Hence we estimate

$$\begin{aligned} \langle \zeta(\vec{x})^3 \rangle &= \int d\log k \frac{8\pi^2}{(2\pi)^6} k^6 B_{\text{eq}}(k) \\ &\simeq \frac{8\pi^2}{(2\pi)^6} k^6 B_{\text{eq}}(k) \mathcal{O}(1), \end{aligned} \quad (D5)$$

where $B_{\text{eq}}(k)$ is the bispectrum evaluated on equilateral triangles. Now we can compare our estimate (D2) with

the precisely computed result using the Green's function approach, that we take from result ((2.8) of [33]),

$$B_{\text{eq}}(k) = \frac{1}{(2\pi)^3} \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle \\ \simeq \frac{3 \times 3 \times 2.8 \times 10^{-7}}{10(2\pi)^2} \frac{H^{12} e^{6\pi\xi}}{\xi^9 \dot{\phi}^6} \frac{1}{k^6}, \quad (\text{D6})$$

where we have used the large ξ estimate

$$f_3(\xi) = \frac{2.8 \cdot 10^{-7}}{\xi^9}. \quad (\text{D7})$$

This last result leads to

$$\langle \zeta(\vec{x})^3 \rangle \simeq \frac{8\pi^2}{(2\pi)^6} k^6 B_{\text{eq}}(k) \simeq 8.2 \times 10^{-12} \frac{H^{12} e^{6\pi\xi}}{\xi^9 \dot{\phi}^6} \quad (\text{D8})$$

which agrees (surprisingly) well with (D2).

In the regime of strong backreaction we can write $\beta \approx -2\pi\xi\alpha \frac{\langle \vec{E} \cdot \vec{B} \rangle}{3H\dot{\chi}}$ and the estimate (D1) directly gives the generalization of (B8)

$$\langle \zeta(\vec{x})^3 \rangle \simeq \frac{1}{4\pi^3 \xi^3}. \quad (\text{D9})$$

Finally we want to convert these results into a value for f_{NL} . We take f_{NL} to be defined via

$$\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \zeta(\vec{k}_3) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \\ \times (2\pi)^4 \frac{3}{10} f_{NL} \Delta_\zeta^4(k) \frac{\sum_i k_i^3}{\prod_i k_i^3}. \quad (\text{D10})$$

This gives

$$f_{NL} = B(\vec{k}_1, \vec{k}_2, \vec{k}_3) \frac{10}{3} \frac{1}{(2\pi)^4} \frac{1}{\Delta_\zeta^4(k)} \frac{\Pi_i k_i^3}{\sum_i k_i^3}, \quad (\text{D11})$$

which for the equilateral case becomes

$$f_{NL}^{\text{eq}} = B_{\text{eq}}(\vec{k}) \frac{10}{3} \frac{1}{(2\pi)^4} \frac{1}{\Delta_\zeta^4(k)} \frac{k^9}{3k^3} \\ = \frac{(2\pi)^6}{8\pi^2} \frac{1}{k^6} \langle \zeta(\vec{x})^3 \rangle \times \frac{10}{3} \frac{1}{(2\pi)^4} \frac{1}{\Delta_\zeta^4(k)} \frac{k^9}{3k^3} \\ = \frac{10}{9} \frac{(2\pi)^2}{8\pi^2} \frac{\langle \zeta(\vec{x})^3 \rangle}{\Delta_\zeta^4(k)}. \quad (\text{D12})$$

In the regime of negligible backreaction we can then take our estimate (D2), and conclude that

$$f_{NL}^{\text{eq}} = \frac{2.8 \cdot 10^{-7}}{\xi^9} \frac{e^{6\pi\xi} \Delta_{\zeta, \text{sr}}^6(k)}{\Delta_\zeta^4(k)}. \quad (\text{D13})$$

This again matches the result obtained in [30, 31] by a more precise computation. (Of course, after that we had found that the expressions for $\langle \zeta(\vec{x})^3 \rangle$ match so well, this is only a consistency check.)

In the regime of strong backreaction, finally, we need to insert (D9) into (D12). Using our power spectrum estimate (B8) we find

$$f_{NL}^{\text{eq}} = \frac{10}{9} \frac{(2\pi)^2}{8\pi^2} \frac{(2\pi\xi)^4}{4\pi^3 \xi^3} = \frac{10}{9} 2\pi\xi \simeq 42 \quad (\text{D14})$$

where we have used that towards the end of inflation we have $\xi \simeq 6$.

Notwithstanding the precise match between (D2) and (D8), there is still an order one factor between the estimate for the three point function (and for f_{NL}) and its precisely computed numerical value. Again: to arrive at (D2) we have used the estimate B10 for $\langle \vec{E} \cdot \vec{B} \rangle$, and to arrive at (D8) we have inserted the large ξ approximation $\frac{2.8 \cdot 10^{-7}}{\xi^9}$ for $f_3(\xi)$. When using precise numerical prescriptions rather than estimates for $\langle \vec{E} \cdot \vec{B} \rangle$ and $f_3(\xi)$ we find that our estimates overshoots the precisely computed f_{NL} by a factor of 9.5 at $\xi = 2.2$ ($N = 60$), and by a factor of 3.8 at $\xi = 3$ ($N \approx 35$).

Again we will not bother introducing a fudge factor to close this gap at some preferred value of ξ . Anyway, when backreaction is large f_{NL} is not a suitable indicator for the amount of non-Gaussianity anymore. In Figure 8 we plot our estimate for a more meaningful quantity: the skewness, which is equivalent to $f_{NL}\zeta$. When backreaction becomes important, it saturates at a value of about one, which a posteriori justifies our approach (36).

Appendix E: Black hole masses

In this appendix we give some details about the derivation of (42) for the black hole mass and about the total number of efoldings enforced by a specific expansion history.

Suppose the universe is radiation dominated right after the end of inflation. Then the expansion proceeds as $a \sim (t/t_0)^{1/2}$, so $H(t) = \frac{1}{2t}$. This regime starts at t_0 , which is not the time since the beginning of Big Bang, but simply the constant $t_0 = \frac{1}{2H}$, where H is the Hubble constant at the end of inflation. We distinguish it from

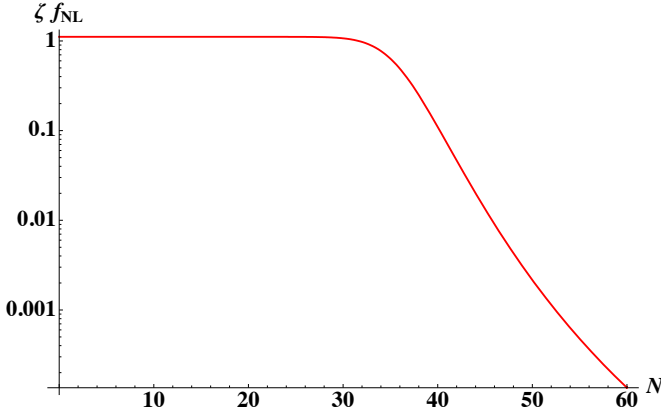


FIG. 8. Evolution of $f_{NL} \times \zeta$ as a function of N , for $\xi[N = 60] = 2.2$

the decreasing $H(t) = \frac{1}{2t}$. The wavelength $l_{t_0} = H^{-1}e^N$ grows as $l_t = H^{-1}(t/t_0)^{1/2}e^N = H^{-1}(2Ht)^{1/2}e^N$. The horizon size $1/H(t) = 2t$ grows and becomes equal to l_t (and black holes form) at

$$2t = H^{-1}(2Ht)^{1/2}e^N$$

i.e. at

$$(2Ht)^{1/2} = (t/t_0)^{1/2} = e^N.$$

In other words, the black holes form after the universe expands by a factor e^N since the end of inflation. The initial energy stored inside the volume $H^{-1}e^N$ was $M_N \simeq 10 e^{3N}$ g, but during this extra expansion it scales down (redshifts) by the factor e^N , so it becomes

$$M_{BH} \simeq 10 e^{2N} g.$$

It should be stressed that specifying the energy density at the end of inflation and at reheating, directly determines the number of efoldings corresponding to any scale (and in particular CMB scales) according to [51]

$$N(k) = 62 - \log \frac{k}{a_0 H_0} - \log \frac{10^{16} \text{GeV}}{V_*^{1/4}} \quad (\text{E1})$$

$$+ \log \frac{V_*^{1/4}}{V_{end}^{1/4}} - \frac{1}{3} \log \frac{V_{end}^{1/4}}{\rho_{reh}^{1/4}},$$

where V_* is the energy density during inflation when the mode k left the horizon, V_{end} is the energy density at the end of inflation, ρ_{reh} is the energy density at reheating and the subscript 0 refers to today's value. Taking for example $\rho_{reh} = V_{end} = m^2 M_p^2/2$ and $V_k = m^2 15^2 M_p^2/2$ with $m = 6 \times 10^6 M_p$ gives $N_{CMB} = N(a_0 H_0) \simeq 64$. We use this value in our discussion of primordial black holes, but since the difference between 60 and 64 changes very little in our numerics, for simplicity we use $N_{CMB} = 60$ in the rest of the paper.

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